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# Casimir effect for thin films in QED

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**Abstract**

We consider the problem of modelling of interaction of thin material films with fields of quantum electrodynamics. Taking into account the basic principles of quantum electrodynamics (locality, gauge invariance, renormalizability), we construct a single model for Casimir-like phenomena arising near the film boundary at distances much larger than the Compton wavelength of the electron. In this region, contribution of fluctuations of Dirac fields is not essential and can be neglected. In the model, the film is presented by a singular background field concentrated on a two-dimensional surface and interacting with the quantum electromagnetic field. All the properties of the film material are described by one dimensionless parameter. For two parallel plane films, the Casimir force appears to be non-universal and dependent on the material property. It can be both attractive and repulsive. In the model, we study the scattering of electromagnetic wave on the plane film, interaction of the plane film with a point charge, homogeneously charged plane and straight line current. Here, besides usual results of classical electrodynamics, the model predicts appearance of anomalous electromagnetic phenomena.

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## 1. Introduction

In 1948, it was shown by Casimir that vacuum fluctuations of quantum fields generate an attraction between two parallel uncharged conducting planes [1]. This phenomenon, called the Casimir effect (CE), has been well investigated with methods of modern experiments [2–4]. The CE is a manifestation of the influence of fluctuations of quantum fields on the level of classical interaction of material objects. Theoretical and experimental investigations of such phenomena became very important for development of micro-mechanics and nanotechnology.

Though there are many theoretical results on the CE [5], the majority of them are received in the framework of several models based not on the quantum electrodynamics (QED) directly. Usually, one assumes that the CE can be investigated in the framework of free massless quantum scalar field theory with fixed boundary conditions or $\delta$-function potentials [6, 7], ignoring restrictions following from gauge invariance, locality and renormalizability of QED. By means of such methods, one can investigate some of the CE properties, but there is no possibility of studying other phenomena generated by interaction of the QED fields with considered classical background within the same model.

An approach for construction of the single QED model for investigation of all peculiar properties of the CE for thin material films was proposed in [8, 9]. We consider its application for the simple case of parallel plane films. We show that gauge invariance, locality and renormalizability considered as basic principles make strong restrictions for constructions of the CE models in QED, which make it possible to reveal new important features of the CE-like phenomena.

## 2. Construction of models

We construct models for interaction of the material film with QED fields on the basis of most general assumptions. We suppose that the film is presented by a singular background (defect) concentrated on the two-dimensional surface. Its interaction with QED fields has a most general form defined by the geometry of the defect and restrictions following from the basic principles of QED (gauge invariance, locality, renormalizability). The locality of interaction means that the action functional of the defect is represented by an integral over the defect surface of the Lagrangian density which is a polynomial function of a spacetime point with respect to fields and derivatives of ones. The coefficients of this polynomial are the parameters defining defect properties. For the quantum field theory (QFT) with singular background, the requirement of renormalizability was analysed by Symanzik in [10]. He showed that in order to keep renormalizability of the model, one needs to add a defect action to the usual bulk action of the QFT model. The defect action must contain all possible terms with non-negative dimensions of parameters and not include any parameters with negative dimensions. In the case of QED, the defect action must also be gauge invariant.

From these requirements, it follows that for a thin film (without charges and currents) whose shape is defined by the equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with the photon field $A_\mu(x)$ reads

$$S_\Phi(A) = \frac{a}{2}\int \varepsilon^{\lambda\mu\nu\rho}\partial_\lambda\Phi(x)A_\mu(x)F_{\nu\rho}(x)\delta(\Phi(x))\,\mathrm{d}x, \tag{1}$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$, $\varepsilon^{\lambda\mu\nu\rho}$ denotes a totally antisymmetric tensor ($\varepsilon^{0123} = 1$) and $a$ is a constant dimensionless parameter. The action (1) is a surface Chern–Simon action [11, 12]. The fermion defect action can be written as

$$S_\Phi(\bar\psi, \psi) = \int \bar\psi(x)[\lambda + u^\mu\gamma_\mu + \gamma_5(\tau + v^\mu\gamma_\mu) + \omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))\,\mathrm{d}x. \tag{2}$$

Here, $\gamma_\mu$, $\mu = 0, 1, 2, 3$, are the Dirac matrices, $\gamma_5 = \mathrm{i}\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu} = \mathrm{i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and $\lambda, \tau, u_\mu, v_\mu, \omega^{\mu\nu} = -\omega^{\nu\mu}$, $\mu, \nu = 0, 1, 2, 3$, are 16 dimensionless parameters.

Expressions (1) and (2) are the most general forms of gauge invariant actions concentrated on the defect surface being invariant with respect to reparametrization of one and not having any parameters with negative dimensions.

We consider in this paper CE-like phenomena arising near the defect boundary at distances much larger than the Compton wavelength of the electron. In this case one can neglect the Dirac fields in QED because of exponential damping of fluctuations of those at much smaller distances ($\sim m_\mathrm{e}^{-1} \approx 10^{-10}$ cm for electron, $\sim m_\mathrm{p}^{-1} \approx 10^{-13}$ cm for proton [8]). Thus, for constructing a model, we can use the action of a free quantum electromagnetic field (photodynamic) with additional defect action (1).

For description of all physical phenomena, it is enough to calculate the generating functional of Green's functions. For the gauge condition $\phi(A) = 0$ it reads

$$G(J) = C \int \mathrm{e}^{\mathrm{i}S(A,\Phi)+\mathrm{i}JA} \delta(\phi(A)) DA, \tag{3}$$

where

$$S(A, \Phi) = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + S_\Phi(A), \tag{4}$$

and the constant $C$ is defined by the normalization condition $G(0)|_{a=0} = 1$. The first term on the right-hand side of (4) is the usual action of a photon field. Along with the defect action, it forms a quadratic in the photon field full action of the system which can be written as $S(A, \Phi) = 1/2 A_\mu K_\Phi^{\mu\nu} A_\nu$. The integral (3) is Gaussian and is calculated exactly as

$$G(J) = \exp\left\{\tfrac{1}{2}\,\mathrm{Tr}\ln(D_\Phi D^{-1}) - \tfrac{1}{2} J D_\phi J\right\},$$

where $D_\Phi$ is the propagator $D_\Phi = \mathrm{i}K_\Phi^{-1}$ of photodynamic with defect in gauge $\phi(A) = 0$, and $D$ is the propagator of the photon field without defect in the same gauge. For the static defect, the function $\Phi(x)$ is time independent, and $\ln G(0)$ defines the Casimir energy.

In order to expose essential features of CE-like phenomena in the constructed model, we calculate the Casimir force (CF) for the simple case of two parallel infinite plane films and study the scattering of electromagnetic waves on the plane defect. We also consider an interaction of the plane film with a straight line current parallel to it and an interaction of the film with a point charge and homogeneous charge distribution on a parallel plane.

## 3. Casimir force

We consider a defect concentrated on two parallel planes $x_3 = 0$ and $x_3 = r$. For this model, it is convenient to use a notation like $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$. The defect action (1) has the form

$$S_{2P} = \frac{1}{2} \int (a_1 \delta(x_3) + a_2 \delta(x_3 - r)) \varepsilon^{3\mu\nu\rho} A_\mu(x) F_{\nu\rho}(x)\, \mathrm{d}x.$$

The defect action $S_{2P}$ was discussed in [13] in substantiation of Chern–Simon type boundary conditions chosen for studies of the Casimir effect in photodynamics. This approach based on boundary conditions is not related directly to the one we present. The defect action (1) is the main point in our model formulation, and no other boundary conditions are used. The action $S_{2P}$ is translationally invariant with respect to coordinates $x_i, i = 0, 1, 2$. The propagator $D_\Phi(x, y)$ is written as

$$D_{2P}(x, y) = \frac{1}{(2\pi)^3} \int D_{2P}(\vec{k}, x_3, y_3)\, \mathrm{e}^{\mathrm{i}\vec{k}(\vec{x}-\vec{y})}\, \mathrm{d}\vec{k},$$

and $D_{2P}(\vec{k}, x_3, y_3)$ can be calculated exactly. Using Latin indices for the components of 4-tensors with numbers 0, 1, 2 and notation

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m/\vec{k}^2, \qquad L^{lm}(\vec{k}) = \epsilon^{lmn3} k_n/|\vec{k}|,$$
$$\vec{k}^2 = k_0^2 - k_1^2 - k_2^2, \qquad |\vec{k}| = \sqrt{\vec{k}^2}$$

($g$ is metrics tensor), one can present the results for the Coulomb-like gauge $\partial_0 A^0 + \partial_1 A^1 + \partial_2 A^2 = 0$ as follows [9]:

$$D_{2P}^{33}(\vec{k}, x_3, y_3) = \frac{-\mathrm{i}\delta(x_3 - y_3)}{|\vec{k}|^2}, \qquad D_{2P}^{l3}(\vec{k}, x_3, y_3) = D_{2P}^{3m}(\vec{k}, x_3, y_3) = 0,$$

$$D_{2P}^{lm}(\vec{k}, x_3, y_3) = \frac{P^{lm}(\vec{k})\mathcal{P}_1(\vec{k}, x_3, y_3) + L^{lm}(\vec{k})\mathcal{P}_2(\vec{k}, x_3, y_3)}{2|\vec{k}|[(1 + a_1 a_2(\mathrm{e}^{2\mathrm{i}|\vec{k}|r} - 1))^2 + (a_1 + a_2)^2]},$$

where

$$\begin{aligned}\mathcal{P}_1(\vec{k}, x_3, y_3) = {} & \left[a_1 a_2 - a_1^2 a_2^2 (1 - \mathrm{e}^{2\mathrm{i}|\vec{k}|r})\right][\mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3| + |y_3 - r|)} + \mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3 - r| + |y_3|)}]\,\mathrm{e}^{\mathrm{i}|\vec{k}|r} \\ & + \left[a_1^2 + a_1^2 a_2^2 (1 - \mathrm{e}^{2\mathrm{i}|\vec{k}|r})\right]\mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3| + |y_3|)} \\ & + \left[a_2^2 + a_1^2 a_2^2 (1 - \mathrm{e}^{2\mathrm{i}|\vec{k}|r})\right]\mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3 - r| + |y_3 - r|)} \\ & - \mathrm{e}^{\mathrm{i}|\vec{k}||x_3 - y_3|}[(1 + a_1 a_2(\mathrm{e}^{2\mathrm{i}|\vec{k}|r} - 1))^2 + (a_1 + a_2)^2], \\ \mathcal{P}_2(\vec{k}, x_3, y_3) = {} & a_1[1 + a_2(a_2 + a_1\,\mathrm{e}^{2\mathrm{i}|\vec{k}|r})]\,\mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3| + |y_3|)} \\ & + a_2[1 + a_1(a_1 + a_2\,\mathrm{e}^{2\mathrm{i}|\vec{k}|r})]\,\mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3 - r| + |y_3 - r|)} \\ & - a_1 a_2 (a_1 + a_2)(\mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3| + |y_3 - r|)} + \mathrm{e}^{\mathrm{i}|\vec{k}|(|x_3 - r| + |y_3|)})\,\mathrm{e}^{\mathrm{i}|\vec{k}|r}.\end{aligned}$$

The energy density $E_{2P}$ of the defect is defined as

$$\ln G(0) = \tfrac{1}{2}\mathrm{Tr}\ln(D_{2P}D^{-1}) = -\mathrm{i}TSE_{2P},$$

where $T = \int \mathrm{d}x_0$ is the duration of the defect, and $S = \int \mathrm{d}x_1\,\mathrm{d}x_2$ is the area of film. It is expressed in an explicit form in terms of the polylogarithm function $\mathrm{Li}_4(x)$ [9]. For identical films with $a_1 = a_2 = a$ it holds that $E_{2P} = 2E_s + E_{\mathrm{Cas}}$, $E_s = \int \ln\sqrt{(1 + a^2)}\frac{\mathrm{d}\vec{k}}{(2\pi)^3}$,

$$E_{\mathrm{Cas}} = -\frac{1}{16\pi^2 r^3}\left\{\mathrm{Li}_4\left(\frac{a^2}{(a + i)^2}\right) + \mathrm{Li}_4\left(\frac{a^2}{(a - i)^2}\right)\right\}.$$

Here, $E_s$ is an infinite constant, which can be interpreted as the self-energy density on the plane, and $E_{\mathrm{Cas}}$ is the energy density of their interaction. The function $\mathrm{Li}_4(x)$ is defined as $\mathrm{Li}_4(x) = \sum_{k=1}^{\infty}\frac{x^k}{k^4} = -\frac{1}{2}\int_0^{\infty} k^2 \ln(1 - x\,\mathrm{e}^{-k})\,\mathrm{d}k$. The force $F_{2P}(r, a)$ between planes is given by

$$F_{2P}(r, a) = -\frac{\partial E_{\mathrm{Cas}}(r, a)}{\partial r} = -\frac{\pi^2}{240 r^4} f(a).$$

The force $F_{2P}$ is repulsive for $|a| < a_0$ and attractive for $|a| > a_0$, $a_0 \approx 1.032\,46$ (see figure 1). For large $|a|$, it is the same as the usual CF between perfectly conducting planes. The model predicts that the maximal magnitude of the *repulsive* $F_{2P}$ is expected for $|a| \approx 0.6$. For two infinitely thick parallel slabs the repulsive CF was also predicted in [14].

Real film has a finite width, and the bulk contributions to the CF for non-perfectly conducting slabs with widths $h_1, h_2$ are proportional to $h_1 h_2$. Therefore, it follows directly from the dimensional analysis that the bulk correction $F_{\mathrm{bulk}}$ to the CF is of the form $F_{\mathrm{bulk}} \approx cF_{\mathrm{Cas}} h_1 h_2 / r^2$, where $F_{\mathrm{Cas}}$ is the CF for perfectly conducting planes, and $c$ is a dimensionless constant. This estimation can be relevant for modern experiments on the CE. For instance, in [4] there were results obtained for parallel metallic surfaces where the width of the layer was about $h \approx 50$ nm and the typical distance $r$ between the surfaces was $0.5\,\mu\mathrm{m} \leqslant r \leqslant 3\,\mu\mathrm{m}$. In that case $3 \times 10^{-4} \leqslant (h/r)^2 \leqslant 10^{-2}$. In [4], authors have fitted the CF between chromium films with function $C_{\mathrm{Cas}}/r^4$. They claim that the value of $C_{\mathrm{Cas}}$ coincides with the known Casimir result within a 15% accuracy. It means that the bulk force

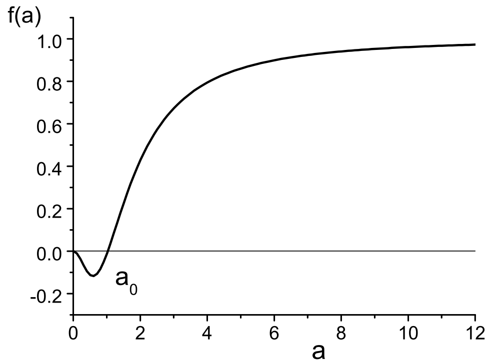


**Figure 1.** Function $f(a)$ determining the Casimir force between parallel planes.

can be neglected, and only surface effects are essential. In our model, the values $a > 4.8$ of the defect coupling parameter $a$ are in good agreement with the results of [4].

Now we study the scattering of classical electromagnetic wave on plane defect and effects generated by the coupling of a plane film with a given classical 4-current.

## 4. Interaction of a film with classical current and electromagnetic waves

The scattering problem is described in our approach by a homogeneous classical equation $K^{\mu\nu}_{2P}A_\nu = 0$ of a simplified model with $a_1 = a, a_2 = 0$. It has a solution in the form of a plane wave. If one defines the transmission (reflection) coefficient as a ratio $K_t = U_t/U_{\rm in}$ ($K_r = U_r/U_{\rm in}$) of the transmitted wave energy $U_t$ (reflected wave energy $U_r$) to the incident wave energy $U_{\rm in}$, then direct calculations give the following result: $K_t = (1 + a^2)^{-1}, K_r = a^2(1 + a^2)^{-1}$. We note the following features of reflection and transmission coefficients. In the limit of an infinitely large defect coupling, these coefficients coincide with coefficients for a perfectly conducting plane. The reflection and transmission coefficients do not depend on the incidence angle.

The classical charge and the wire with a current near the defect plane are modelled by the appropriately chosen 4-current $J$ in (3). The mean vector potential $\mathcal{A}_\mu$ generated by $J$ and the plane $x_3 = 0$, with $a_1 = a$, can be calculated as

$$\mathcal{A}^\mu = -\mathrm{i}\frac{\delta G(J)}{\delta J_\mu}\bigg|_{a_1=a,a_2=0} = \mathrm{i}D^{\mu\nu}_{2P}J_\nu\big|_{a_1=a,a_2=0}. \tag{5}$$

Using notation $\mathcal{F}_{ik} = \partial_i\mathcal{A}_k - \partial_k\mathcal{A}_i$, one can present electric and magnetic fields as $\vec{E} = (\mathcal{F}_{01}, \mathcal{F}_{02}, \mathcal{F}_{03})$, $\vec{H} = (\mathcal{F}_{23}, \mathcal{F}_{31}, \mathcal{F}_{12})$. For charge $e$ at the point $(x_1, x_2, x_3) = (0, 0, l)$, $l > 0$, the corresponding classical 4-current is

$$J_\mu(x) = 4\pi e\delta(x_1)\delta(x_2)\delta(x_3 - l)\delta_{0\mu}.$$

In virtue of (5), the mean vector potential $\mathcal{A}^\mu(x)$ is independent of $x_0$ and the electric field in the considered system is defined by the potential

$$\mathcal{A}_0(x_1, x_2, x_3) = \frac{e}{\rho_-} - \frac{a^2}{a^2+1}\frac{e}{\rho_+},$$

where $\rho_+ \equiv \sqrt{x_1^2 + x_2^2 + (|x_3| + l)^2}$, $\rho_- \equiv \sqrt{x_1^2 + x_2^2 + (x_3 - l)^2}$. The electric field $\vec{E} = (E_1, E_2, E_3)$ is of the form

$$E_1 = \frac{ex_1}{\rho_-^3} - \frac{a^2}{a^2+1}\frac{ex_1}{\rho_+^3},$$
$$E_2 = \frac{ex_2}{\rho_-^3} - \frac{a^2}{a^2+1}\frac{ex_2}{\rho_+^3},$$
$$E_3 = \frac{e(x_3 - l)}{\rho_-^3} - \frac{a^2\epsilon(x_3)}{a^2+1}\frac{e(|x_3| + l)}{\rho_+^3}.$$

Here, $\epsilon(x_3) \equiv x_3/|x_3|$. We see that for $x_3 > 0$, the field $\vec{E}$ coincides with the field generated in usual classical electrostatic by a charge $e$ placed at a distance $l$ from an infinitely thick slab with dielectric constant $\epsilon = 2a^2 + 1$.

Because $\mathcal{A}^\mu(x) \neq 0$ for $\mu = 1, 2, 3$, the defect also generates a magnetic field $\vec{H} = (H_1, H_2, H_3)$:

$$H_1 = \frac{eax_1}{(a^2+1)\rho_+^3}, \qquad H_2 = \frac{eax_2}{(a^2+1)\rho_+^3}, \qquad H_3 = \frac{ea(|x_3| + l)}{(a^2+1)\rho_+^3}.$$

It is an anomalous field which does not arise in classical electrostatics. Its direction depends on the sign of $a$. In a similar way, one can calculate the fields generated by interaction of the film and the charged plane $x_3 = l$, presented by the classical current

$$J_\mu(x) = 4\pi\sigma\delta(x_3 - l)\delta_{0\mu},$$

Here, $\sigma$ is the charge density. In this case, it holds that

$$E_1 = E_2 = 0 = H_1 = H_2 = 0,$$
$$E_3 = 2\pi\sigma\left(\epsilon(x_3 - l) - \epsilon(x_3)\frac{a^2}{a^2+1}\right),$$
$$H_3 = 2\pi\sigma\frac{a}{a^2+1}.$$

Thus, in the considered system, there is only one component of fields $\vec{E}$, $\vec{H}$ dependent on $l$. It is $E_3$. For $l \to \mp\infty$

$$E_3 = 2\pi\sigma\big(\pm 1 - \frac{\epsilon(x_3)a^2}{a^2+1}\big),$$

and for

$$l = 0 \qquad E_3 = \frac{2\pi\sigma\epsilon(x_3)}{a^2+1}.$$

It is important to note that anomalous fields arise because the space parity is broken by the action (4), and they are generated in (5) by the $L\mathcal{P}_2$-term of propagator $D_{2P}$.

A current with density $j$ flowing in the wire along the $x_1$-axis is modelled by

$$J_\mu(x) = 4\pi j\delta(x_3 - l)\delta(x_2)\delta_{\mu 1}.$$

For the magnetic field from (5) one obtains in the region $x_3 > 0$ the usual results of classical electrodynamics for the current parallel to the infinitely thick slab with permeability $\mu = (2a^2 + 1)^{-1}$. There is also an anomalous electric field $\vec{E} = (0, E_2, E_3)$:

$$E_2 = \frac{2ja}{a^2+1}\frac{x_2}{\tau^2}, \qquad E_3 = \frac{2ja}{a^2+1}\frac{|x_3| + l}{\tau^2},$$

where $\tau = (x_2^2 + (|x_3| + l)^2)^{\frac{1}{2}}$. Comparing the formulae $\epsilon = 2a^2 + 1$ and $\mu = (2a^2 + 1)^{-1}$ for the parameter $a$, we obtain the relation $\epsilon\mu = 1$. It holds for a thick slab material interaction of which with a point charge and current in classical electrodynamics was compared with the results for a thin film of our model. The speed of light in this hypothetical material is equal to that in the vacuum. From the physical point of view it could be expected that because interaction of the film with the photon field is a surface effect which cannot generate the bulk phenomena like decreasing the speed of light in the considered slab. With this argument, it seems to be not surprising that the reflection coefficient of electromagnetic wave in our model is independent of the incidence angle, since by $\epsilon\mu = 1$ it holds for Fresnel formulae too.

The relation $\epsilon\mu = 1$ is not new in the context of the Casimir theory. It was first introduced by Brevik and Kolbenstvedt [15], who calculated the Casimir surface force density on the sphere. Only under this condition, a contact term turns out to be zero [15]. It has been investigated in a number of subsequent papers. In our approach, this condition arises naturally because we have only one parameter $a$ that must describe both magnetostatic and electrostatic properties of the film.

The essential property of interaction of films with classical charge and current is the appearance of anomalous fields. These fields are suppressed in respect of usual ones by a factor $a^{-1}$ and they vanish in the case of a perfectly conducting plane. Magnetoelectric (ME) films are good candidates for detecting anomalous fields and non-ideal CE. The generic example of ME crystals is $Cr_2O_3$ [16]. It is important to note that for ME films, the Lifshitz theory of CE is not relevant, but they can be studied in our approach.

## 5. Conclusion

The main results of our study on the CE for thin films in the QED are as follows. We have shown that if the CF holds true for a thin material film, then an interaction of this film with the QED fields can be modelled by photodynamics with the defect action (1) obtained by most general assumptions consistent with locality, gauge invariance and renormalizability of model. Thus, basic principles of QED were essential in our studies of the CE. These principles make it possible to expose new peculiarities of the physics of macroscopic objects in QED and must be taken into account for construction of the models. For plane films, we have demonstrated that the CF is not universal and depends on properties of the material represented by the parameter $a$. For $a \to \infty$, one can obtain the CF for ideal conducting planes. In this case, the model coincides with photodynamics considered in [17] with the boundary condition $\epsilon^{ijk3}F_{jk} = 0$ ($i = 0, 1, 2$) orthogonal to the $x_3$-axis planes. For sufficiently small $a$, the CF appears to be repulsive. Interaction of plane films with charges and currents generates anomalous magnetic and electric fields which do not arise in classical electrodynamics. The ME materials could be used for observation of phenomena predicted by our model. We hope that the obtained theoretical results can be proven by modern experimental methods.

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